J/ψ electromagnetic production associated with light hadrons at B factories

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Abstract

The electromagnetic productions of J/ψ associated with light hadrons(LH) and leptonic pairs($\mu^+\mu^-$, $\tau^+\tau^-$) at B factories are studied. We find that the direct electromagnetic production cross sections of $J/\psi(\psi(2S))$ associated with light hadrons is about 0.10(0.04) pb. The direct production cross sections of $J/\psi(\psi(2S))$ associated with $\mu^+\mu^-$ is about 0.056(0.020) pb. If we include the contributions from $\psi(2S)$ decay, we can get the prompt cross section $\sigma[e^+e^- \to J/\psi + \mu^+\mu^-] = (0.068 \pm 0.002)$ pb, about $(16 \pm 5)\%$ of the Belle data $\sigma[e^+e^- \to J/\psi + X_{non-c\bar{c}}] = (0.43 \pm 0.09 \pm 0.09)$ pb, meanwhile the $e^+e^- \to J/\psi + \tau^+\tau^-$ process only contributes 2%. The prompt cross section $\sigma[e^+e^- \to J/\psi + Light\ Hadrons] = (0.121 \pm 0.005)$ pb is about $(28 \pm 8)\%$ of the Belle data.

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I. INTRODUCTION

Heavy quarkonium physics is an important ground to test quantum chromodynamics (QCD) both perturbatively and non-perturbatively. Non-relativistic quantum chromodynamics (NRQCD) factorization approach[1] has achieved a series of successes in describing heavy quarkonium production and annihilation decay. However, there are still some predictions which are less satisfactory. For more details, see a concise review[2]. Among the problematic comparisons with experiment, the large discrepancy between theoretical predictions and experimental data on the charmonium production in e^+e^- annihilation has drawn much attention recently.

In recent years, the B factories has provided systematic measurements on charmonium production. Some results are puzzling, because of the large gap between the measurements and the theoretical predictions. For example, the cross section $\sigma(e^+ + e^- \to J/\psi + \eta_c)$, measured by Belle Collaboration[3] and Babar Collaboration[4], is almost one order-of-magnitude larger than the leading-order(LO) predictions[5–7]. By introducing the QCD perturbative correction[8, 9], and in combination with relativistic correction[10, 11], this discrepancy was largely resolved. Besides the challenges in the exclusive process, the large ratio of J/ψ production associated with charmed hadrons is also confusing, which was measured by Belle[3]

$$R_{c\bar{c}} = \frac{\sigma[e^+e^- \to J/\psi + c\bar{c}]}{\sigma[e^+e^- \to J/\psi + X]} = 0.59^{+0.15}_{-0.13} \pm 0.12. \tag{1}$$

In contrast to the leading-order NRQCD predictions, this ratio is only about 0.1[12–14]. The next-leading-order(NLO) QCD corrections were also introduced in J/ψ inclusive production to resolve the discrepancy between experimental measurements and LO calculations. The NLO QCD corrections to $e^+e^- \to J/\psi c\bar{c}$ process enhance the cross section with a K factor of about 1.8[15, 16], and only about 20 percent for $e^+e^- \to J/\psi gg$ process [17, 18]. So the discrepancy is greatly alleviated. To check what role does the color-octet process play, the NLO QCD corrections to color-octet J/ψ inclusive production was calculated[19]. Combining the relativistic corrections to $e^+e^- \to J/\psi gg$[20], it may imply that the values of color-octet matrix elements are much smaller than the expected ones which are estimated by using the naive velocity scaling rules.

Another interesting topic is the $e^+e^- \to J/\psi + X_{non-c\bar{c}}$ production. Most recently, the prompt J/ψ production in association with charmed and non-charmed final particles was

measured[21]

$$\begin{aligned}\sigma(e^+e^- \to J/\psi + X) &= (1.17 \pm 0.02 \pm 0.07)pb,\\ \sigma(e^+e^- \to J/\psi + c\bar{c}) &= (0.74 \pm 0.08^{+0.09}_{-0.08})pb\\ \sigma(e^+e^- \to J/\psi + X_{non-c\bar{c}}) &= (0.43 \pm 0.09 \pm 0.09)pb. \end{aligned} \tag{2}$$

These processes are investigated in Ref.[20, 22], the results show that including both the $O(\alpha_s)$ radiative correction and the $O(v^2)$ relativistic correction, the color-singlet contribution to $e^+e^- \to J/\psi gg$ has saturated the latest observed cross section $e^+e^- \to J/\psi + X_{non-c\bar{c}}$ measured by Belle.

Aside from the above QCD process, the pure QED process should also be considered[23]. Especially, in our paper, we calculate virtual-photon-associated production $\sigma(e^+e^- \to J/\psi\gamma^*) \times B(\gamma^* \to l\bar{l}(or\ Light\ Hadrons))$. The rest of the paper is organized as follows. In Section II, we will give the formulations of $e^+e^- \to J/\psi + \mu^+\mu^-$. In Section III , the QED production of $e^+e^- \to J/\psi + LH$ is discussed. In section IV, we will give the numerical results and discussion. Finally we summarize our results in section V.

II. THE FORMULATIONS OF $e^+e^- \to J/\psi + \mu^+\mu^-$

In NRQCD factorization scheme, the cross section of $e^+e^- \to J/\psi + \mu^+\mu^-$ can be described as follows

$$\begin{aligned}&\mathcal{A}(e^+(k_1)e^-(k_2) \to J/\psi(2p) + \mu^+(p_1) + \mu^-(p_2))\\ &\qquad = \sqrt{C_S}\sum_{L_zS_z}\sum_{s_1s_2}\sum_{jk}\langle\frac{1}{2}s_1;\frac{1}{2}s_2 \mid SS_z\rangle\langle LL_z;SS_z \mid JJ_z\rangle\langle 3j;\bar{3}k \mid 1\rangle \times\\ &\qquad\quad \mathcal{A}\big(e^+e^- \to c_j^{s1}(p) + \bar{c}_k^{s2}(p) + \mu^+(p_1) + \mu^-(p_2)\big)\end{aligned} \tag{3}$$

where $\langle 3j;\bar{3}k \mid 1\rangle = 1/\sqrt{3}$, $\langle s_1;s_2 \mid SS_z\rangle$, $\langle LL_z;SS_z \mid JJ_z\rangle$ are respectively the color-SU(3), spin-SU(2), and angular momentum Clebsch-Gordan coefficients for $c\bar{c}$ pairs projecting out appropriate bound states. $\mathcal{A}(e^+e^- \to c_j(p) + \bar{c}_k(p) + \mu^+(p_1) + \mu^-(p_2))$ is the scattering amplitude for $c\bar{c}$ production. The coefficient C_S can be related to the radial wave function of the bound state and reads

$$C_S = \frac{1}{4\pi} \mid R_S(0) \mid^2 . \tag{4}$$

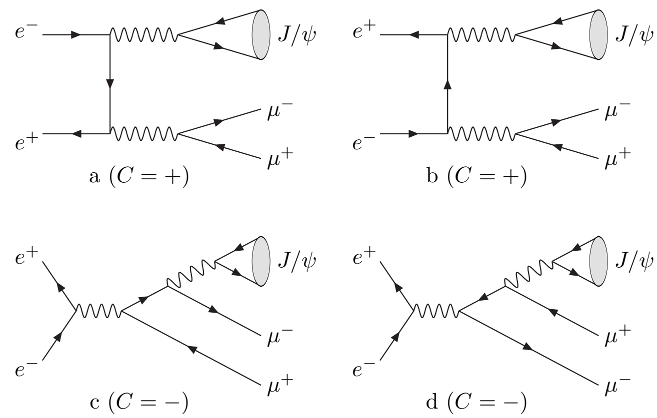

FIG. 1. The Feynman diagrams of $e^+e^- \to J/\psi\mu^+\mu^-$.

We introduce the spin projection operators $P_{SS_z}(p,q)$ as

$$P_{SS_z}(p,q) \equiv \sum_{s_1 s_2} \langle s_1; s_2 | S S_z \rangle v(p-q; s_1) \bar{u}(p+q; s_2). \tag{5}$$

Expanding $P_{SS_z}(p,q)$ in terms of the relative momentum q, we get the projection operators at leading term of q, which will be used in our calculation, as follows

$$P_{1S_z}(p,0) = \frac{1}{\sqrt{2}} \not\epsilon^*(S_z)(\not p + M/2). \tag{6}$$

where M is the mass of the charmonium. It is two times of charm quark mass m in the non-relativistic approximation. The polarized cross section can be calculated by defining the longitudinal polarization vector as follows

$$\epsilon^\mu_L(p) = \frac{2p^\mu}{M} - \frac{M n^\mu}{2n \cdot p}, \tag{7}$$

where $4p^2 = M^2$ and $n^\mu = (1, -\vec{p}/\mid \vec{p}\mid)$.

The Feynman diagrams of $e^+e^- \to J/\psi\mu^+\mu^-$ are shown in Fig.1. The process of final states with plus charge parity is depicted in diagram Fig.1(a,b) and denoted as $C = +$. The process of final states with minus charge parity is depicted in diagram Fig.1(c,d) and denoted as $C = -$. The $C = +$ process is dominant, and the $C = -$ process is suppressed by a factor of

$$f \sim \ln\left(\frac{s}{4m_\mu^2}\right)\frac{s}{M^2}, \tag{8}$$

here the logarithm term come from quasi-collinear divergence with $m_\mu^2 \ll s$. The s/M^2 term come the photon propagator. One can get $f \sim 10^{-2}$ for $e^+e^- \to J/\psi\mu^+\mu^-$ at $\sqrt{s} = 10.6$ GeV for B factories.

III. THE QED PRODUCTION OF $e^+e^- \to J/\psi + LH$

Similar to the $e^+e^- \to J/\psi\mu^+\mu^-$ process, the fragment process is also dominant in $e^+e^- \to J/\psi + Light\ Hadrons$ process. The fragment process can be considered as $e^+e^- \to \gamma^*\gamma^*$, then the virtual photons fragment into J/ψ and light hadrons respectively. By using the approach of the calculation of the hadronic part of the muon $g-2$[24], this process can be described as

$$\frac{\mathrm{d}\sigma^{QED}[e^+e^- \to J/\psi + LH]}{\mathrm{d}m_{LH}^2} \sim \frac{\mathrm{d}\sigma[e^+e^- \to J/\psi + \mu^+\mu^-]}{\mathrm{d}m_{\mu^+\mu^-}^2} \times R^{had}(m_{\mu^+\mu^-}^2)\Bigg|_{m_{\mu^+\mu^-}=m_{LH}}, \quad (9)$$

where

$$R^{had}(\Lambda^2) = \frac{\sigma[e^+e^- \to hadrons]}{\sigma[e^+e^- \to \mu^+\mu^-]}\Bigg|_{m_{e^+e^-}^2=\Lambda^2}, \quad (10)$$

because of the contribution of the $C = -$ process is negligible, after subtracting the effect of $\gamma^* \to c\bar{c}$ from $R^{had}(\Lambda^2)$ by a naive factor $4/3\Theta(\Lambda - M_{c\bar{c}-The})$, we get

$$\begin{aligned}&\frac{\mathrm{d}\sigma^{QED}[e^+e^- \to J/\psi + LH]}{\mathrm{d}m_{LH}^2}\\ = &\frac{\mathrm{d}\sigma[e^+e^- \to J/\psi + \mu^+\mu^-]}{\mathrm{d}m_{\mu^+\mu^-}^2} \times \left[R^{had}(m_{\mu^+\mu^-}^2) - \frac{4}{3}\Theta(m_{\mu^+\mu^-}^2 - M_{c\bar{c}-The}^2)\right]\Bigg|_{m_{\mu^+\mu^-}=m_{LH}} \end{aligned} \quad (11)$$

where Θ is step function, $M_{c\bar{c}-The}$ should be correspond to the $c\bar{c}$ threshold of $M_{J/\psi}$, $2M_D$, etc. The uncertainties should be discussed in the next section.

IV. NUMERICAL RESULT

In numerical calculations, the parameters are selected as [25]:

$$\begin{aligned} &M_\mu = 0.1057GeV, \quad M_{J/\psi} = 3.0969GeV, \quad \sqrt{s} = 10.6GeV,\\ &M_\tau = 1.7768GeV, \quad M_{\psi(2S)} = 3.686GeV, \quad \alpha = 1/132.33 \end{aligned} \quad (12)$$

The table of $R^{had}(\lambda^2)$ have been given in Ref.[24]. We construct an interpolation of $R^{had}(\Lambda^2)$ corresponding to the table with first-order interpolation and setting $R^{had}(\Lambda^2) = 0$ when Λ is less than λ_{min} in the table.

The wave function at the origin can be extracted from the leptonic width $\Gamma(V \to l^+l^-)$

$$|R_S(0)|^2 = \frac{m_V^2}{4e_Q^2\alpha^2}\Gamma[V \to e^+e^-]. \quad (13)$$

The leptonic width of charmonium decays into e^+e^- has been given in Ref.[25]

$$\Gamma[J/\psi \to e^+e^-] = 5.55 \pm 0.14keV,$$
$$\Gamma[\psi(2S) \to e^+e^-] = 2.38 \pm 0.04keV. \tag{14}$$

When we calculate the prompt production cross sections of J/ψ, we take into account the feeddown contribution from $\psi(2S)$ by $B[\psi(2S) \to J/\psi + X] = (57.4 \pm 0.9)\%$[25] and ignore the contribution of the other charmonium. Then we can get the direct production cross section of $J/\psi(\psi(2S))$ associated with $\tau^+\tau^-$ and $\mu^+\mu^-$ at B factories as

$$\sigma^{direct}[e^+e^- \to J/\psi + \mu^+\mu^-] = 56 \pm 2\ fb$$
$$\sigma^{direct}[e^+e^- \to J/\psi + \tau^+\tau^-] = 6.4 \pm 0.2\ fb \tag{15}$$

and

$$\sigma^{direct}[e^+e^- \to \psi(2S) + \mu^+\mu^-] = 20 \pm 1\ fb$$
$$\sigma^{direct}[e^+e^- \to \psi(2S) + \tau^+\tau^-] = 1.8 \pm 0.1\ fb \tag{16}$$

Most of the uncertainties come from the uncertainty of leptonic width in Eq.(14). The others come from the effect of fine structure constant α and higher order QED corrections and so on. The QCD corrections have been taken into account in the leptonic width of $J/\psi(\psi(2S))$.

The cross sections for $C = -$ process is only 1.6%(1.0%) of that for $C = +$ process in $J/\psi(\psi(2S))$ production associated with $\mu^+\mu^-$. And the ratio is about 6.0%(3.9%) in $J/\psi(\psi(2S))$ production associated with $\tau^+\tau^-$. These results are in agreement with the estimation in Eq.(8). So the contribution of $C = -$ process can be ignored in the calculation of electromagnetic $J/\psi(\psi(2S))$ production associated with light hadrons. Finally, we get the direct production cross section of $J/\psi(\psi(2S))$ associated with light hadrons at B factories as

$$\sigma^{direct}_{QED}[e^+e^- \to J/\psi + LH] = 100 \pm 5\ fb$$
$$\sigma^{direct}_{QED}[e^+e^- \to \psi(2S) + LH] = 36 \pm 1\ fb \tag{17}$$

here we choose $M_{c\bar{c}-The} = 2M_D$ in Eq.(11). If we choose $M_{c\bar{c}-The} = M_{J/\psi}$, there is a difference of $-1fb$. So the uncertainties from $M_{c\bar{c}-The}$ can be ignored. Most of the uncertainties come from R^{had} and the leptonic decay width.

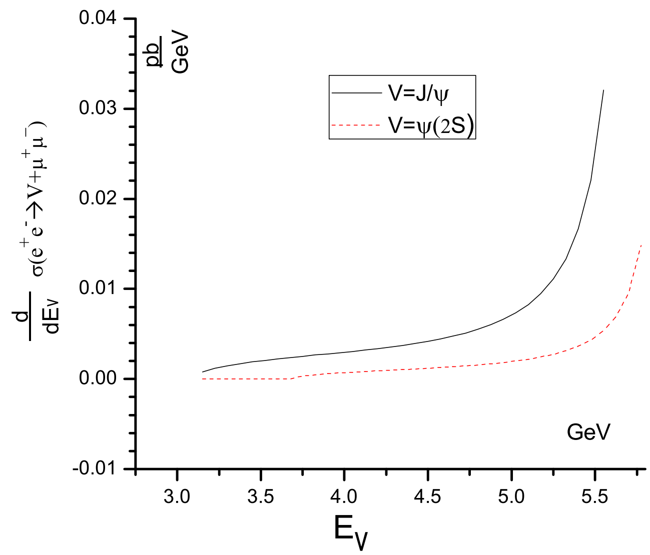

FIG. 2. The J/ψ and $\psi(2S)$ energy spectra of direct production processes $e^+e^- \to V + \mu^+\mu^-$ $(V = J/\psi, \psi(2S))$.

The energy distributions of direct $J/\psi(\psi(2S))$ production from the processes $e^+e^- \to J/\psi(\psi(2S)l\bar{l}$ and $e^+e^- \to J/\psi(\psi(2S))LH$ are shown in Fig.2 and Fig.3, respectively. Unfortunately, the endpoint peak was not measured in Ref.[21]. The polarization of $J/\psi(\psi(2S))$ direct production for $e^+e^- \to V + \mu^+\mu^-$ $(V = J/\psi, \psi(2S))$ process as a function of the energy of J/ψ is shown in Fig.4. The polarization of J/ψ production associated with light hadrons is similar to the results of $e^+e^- \to V + \mu^+\mu^-$ process. The angular distributions of direct $J/\psi(\psi(2S))$ production from the processes $e^+e^- \to V + \mu^+\mu^-(V = J/\psi, \psi(2S))$ and $e^+e^- \to 2\gamma* \to V + LH$ $(V = J/\psi, \psi(2S))$ are shown in Fig.5 and Fig.6, respectively.

Now we give the prompt production cross sections of electromagnetic J/ψ production associated with leptonic pairs and light hadrons as

$$\begin{aligned}\sigma_{QED}^{prompt}[e^+e^- \to J/\psi + \mu^+\mu^- + X] &= 68 \pm 2\ fb\\ \sigma_{QED}^{prompt}[e^+e^- \to J/\psi + \tau^+\tau^- + X] &= 7.4 \pm 0.1\ fb\\ \sigma_{QED}^{prompt}[e^+e^- \to J/\psi + LH] &= 121 \pm 5\ fb. \end{aligned} \tag{18}$$

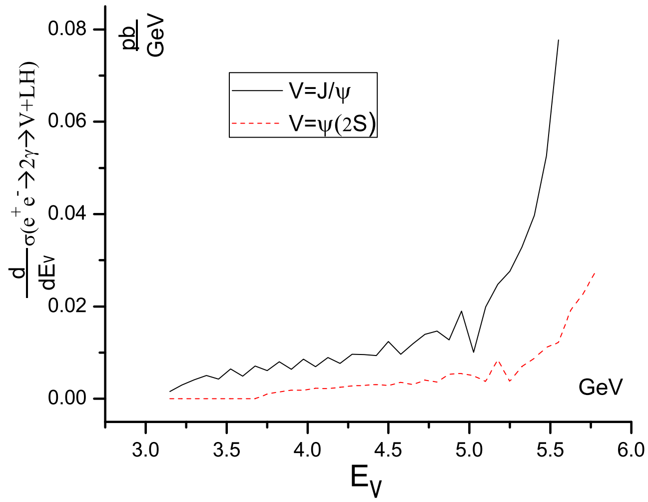

FIG. 3. The J/ψ and $\psi(2S)$ energy spectra of direct production processes $e^+e^- \to 2\gamma \to V + LH$ $(V = J/\psi, \psi(2S))$.

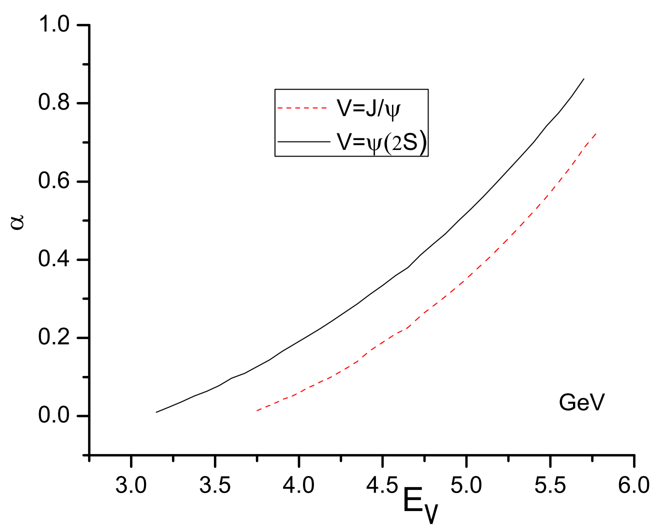

FIG. 4. The helicities of J/ψ and $\psi(2S)$ direct production processes $e^+e^- \to V + \mu^+\mu^-$ $(V = J/\psi, \psi(2S))$ as a function of the energy of J/ψ and $\psi(2S)$.

V. SUMMARY

In summary, the electromagnetic productions of J/ψ associated with light hadrons(LH) and leptonic pairs($\mu^+\mu^-$, $\tau^+\tau^-$) at B factories are studied. We find that the direct electromagnetic production cross sections of $J/\psi(\psi(2S))$ associated with light hadrons is about 0.10(0.04) pb. The direct production cross section of $J/\psi(\psi(2S))$ associated with $\mu^+\mu^-$

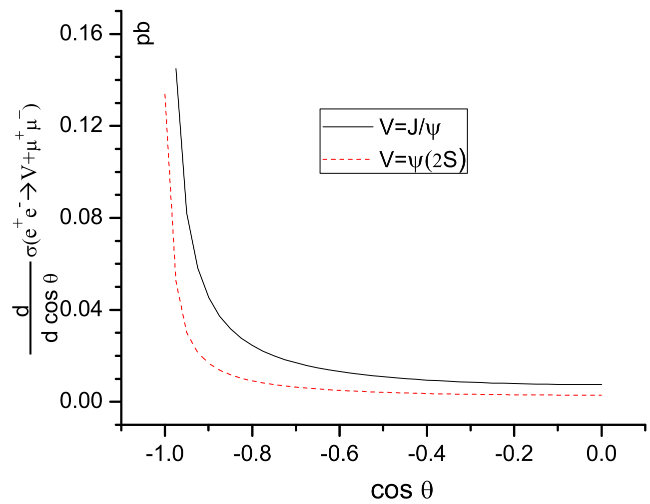

FIG. 5. The angular distributions of direct production processes $e^+e^- \to V + \mu^+\mu^-(V = J/\psi, \psi(2S))$. Here θ is the angle between $J/\psi(\psi(2S))$ momentum and beam.

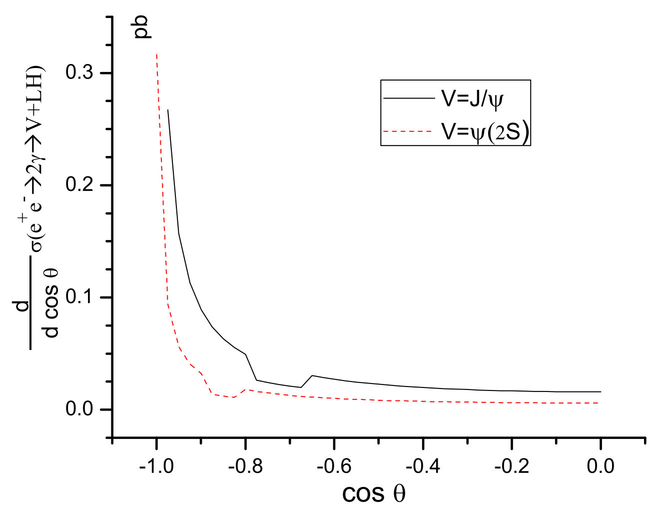

FIG. 6. The angular distributions of direct production processes $e^+e^- \to 2\gamma \to V + LH$ $(V = J/\psi, \psi(2S))$. Here θ is the angle between $J/\psi(\psi(2S))$ and beam.

is about 0.056(0.020) pb. If we include the contribution from $\psi(2S)$ decay, we can get the prompt cross section $\sigma[e^+e^- \to J/\psi + \mu^+\mu^- + X] = (68 \pm 2)$ pb, about $(16 \pm 5)\%$ of the Belle data $\sigma[e^+e^- \to J/\psi + X_{non-c\bar{c}}] = (0.43 \pm 0.09 \pm 0.09)$ pb, meanwhile the $e^+e^- \to J/\psi + \tau^+\tau^-$ process only contributes 2%. The prompt cross section $\sigma[e^+e^- \to J/\psi + Light\ Hadrons] = (0.121 \pm 5)$ fb is about $(28 \pm 8)\%$ of the Belle data. Unfortunately, the endpoint peak of energy distribution for J/ψ electromagnetic production associated with leptonic pairs and

light hadrons was not measured in Ref.[21]. The polarization of J/ψ electromagnetic production associated with light hadrons is transversal, while the polarization of J/ψ inclusive production associated with light hadrons from QCD process is longitudinal[22]. We also notify that the charge parity of final states is plus for the QED process calculated in our paper. And it is minus for color singlet process $e^+e^- \to J/\psi + gg$ and color octet process $e^+e^- \to J/\psi + g$.

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